Head Office: 2nd Floor, Grand Plaza, Fraser Road, Dak Bunglow, Patna - 01

JEE Main 2023 (Memory based)

31 January 2023 - Shift 2

Answer & Solutions

MATHEMATICS

1. Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. There is a vector $\vec{u}$ such that $\vec{u} \times \vec{a} = \vec{b} \times \vec{c}$ and $\vec{u} \cdot \vec{a} = 0$. find $25|\vec{u}|^2$

A. 560

B. $\frac{925}{7}$

C. 446

D. 330

Answer (B)

Solution:

$|\vec{u} \times \vec{a}|^2 + |\vec{u} \cdot \vec{a}|^2 = |\vec{u}|^2|\vec{a}|^2$

$\left|\vec{b} \times \vec{c}\right|^2 + 0 = |\vec{u}|^2 \times 14$

$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 1 & 2 & 2 \end{vmatrix}$

$\vec{b} \times \vec{c} = \hat{\imath}(-8) - \hat{\jmath}(-1) + \hat{k}(3)$

$\vec{b} \times \vec{c} = -8\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$|\vec{b} \times \vec{c}| = \sqrt{74}$

$\Rightarrow \left(\sqrt{74}\right)^2 = |\vec{u}|^2 \times 14$

$\Rightarrow 25|\vec{u}|^2 = \frac{74}{14} \times 25$

$= \frac{925}{7}$

2. The range of $y = \frac{x^2+2x+1}{x^2+8x+1}$ in the domain of function is :

A. $\left(-\infty, -\frac{2}{3}\right] \cup [2, \infty)$

B. $(-\infty, 0] \cup \left[\frac{2}{5}, \infty\right)$

C. $(-\infty, \infty)$

D. $\left(-\infty, -\frac{2}{5}\right] \cup [1, \infty)$

Answer (B)

Solution:

$y = \frac{x^2+2x+1}{x^2+8x+1}$

$\Rightarrow x^2(y-1) + x(8y-2) + y - 1 = 0,\ x \in R$

If $y \neq 1$

$D \geq 0$

$\Rightarrow 4(4y-1)^2 - 4(y-1)(y-1) \geq 0$

$\Rightarrow (4y-1)^2 - (y-1)^2 \geq 0$

$\Rightarrow \big(4y - 1 - (y-1)\big)\big(4y - 1 + (y-1)\big) \geq 0$

$\Rightarrow (3y)(5y-2) \geq 0$

$y \in (-\infty, 0] \cup \left[\frac{2}{5}, \infty\right) - \{1\}$

If $y = 1$

$6x = 0 \Rightarrow x = 0$

$\therefore y \in (-\infty, 0] \cup \left[\frac{2}{5}, \infty\right)$

3. If $a,\ b \in I$ and relation R_1 is defined as $a^2 - b^2 \in I$ and relation R_2 is defined as $2 + \frac{a}{b} > 0$, then:

A. R_1 is symmetric but R_2 is not
B. R_2 is symmetric but R_1 is not
C. R_1 and R_2 are both symmetric
D. R_1 and R_2 are both transitive

Answer (A)

Solution:

$R_1 \to a^2 - b^2 \in I$

As $a,\ b \in Z$ if $a^2 - b^2 \in Z$ then $b^2 - a^2 \in Z$

Also $a^2 - b^2 \in Z$ & $b^2 - c^2 \in Z \Rightarrow a^2 - c^2 \in Z$

$\therefore R_1$ is symmetric as well as transitive.

$R_2 \to 2 + \frac{a}{b} > 0$

Let $a = -1$ and $b = 10$

So, for $(a,\ b) \in R_2 \Rightarrow 2 + \frac{a}{b} > 0$

Now, for $(b, a) \in R_2 \Rightarrow 2 + \frac{b}{a} = 2 - 10 < 0$

$2 + \frac{b}{a} \notin R_2 \Rightarrow R_2$ is NOT symmetric.

4. If $\int \frac{xdx}{\sqrt{x^2+x+2}} = Af(x) + Bg(x) + C$ where C is constant of integration, then $A + 2B$ is equal to:

A. 1
B. 0
C. -1
D. -2

Answer (B)

Solution:

Let $x = \alpha(2x+1) + \beta$
$\alpha = \frac{1}{2},\ \beta = -\frac{1}{2}$
$\int \frac{xdx}{\sqrt{x^2+x+2}} = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+2}}dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2+x+2}}$
$= \frac{1}{2} \times 2\sqrt{x^2+x+2} - \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2 + \frac{7}{4}}}dx + C$
$= \sqrt{x^2+x+2} - \frac{1}{2}\ln\left|x + \frac{1}{2} + \sqrt{x^2+x+2}\right| + C$

$I = Af(x) + Bg(x) + C$
$\Rightarrow A = 1,\ B = -\frac{1}{2}$
$\therefore A + 2B = 0$

5. $\lim_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$ is equal to:

A. 27
B. $\frac{27}{2}$
C. 18
D. 6

Answer (A)

Solution:

$$\lim_{x\to\infty} \frac{\left(\sqrt{3x^2+1}+\sqrt{3x^2-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6} = \lim_{x\to\infty} \frac{x^6\left(\sqrt{3+\frac{1}{x^2}}+\sqrt{3-\frac{1}{x^2}}\right)^6}{x^6\left[\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right]}$$
$$= \frac{(\sqrt{3}+\sqrt{3})^6}{(1+\sqrt{1})^6+(1-\sqrt{1})^6}$$
$$= \frac{(2\sqrt{3})^6}{2^6}$$
$$= \left(\sqrt{3}\right)^6$$
$$= 27$$

6. Foot of perpendicular from origin to a plane which cuts the coordinate axes at A, B, C is $(2,0,4)$. Volume of tetrahedron $OABC$ is $144\ m^3$. Which of the following point does not lie on plane?

A. $(2,2,4)$
B. $(0,3,4)$
C. $(1,1,5)$
D. $(5,5,1)$

Answer (B)

Solution:

Equation of required plane :

$2(x-2) + a(y-a) + 4(z-4) = 0$

$\Rightarrow 2x + ay + 4z = 20 + a^2$

$A\left(10 + \frac{a^2}{2}, 0,0\right),\ B\left(0, \frac{20+a^2}{a}, 0\right), C\left(0,0, \frac{20+a^2}{4}\right)$

Volume of tetrahedron $\frac{1}{6}[\vec{a}\quad \vec{b}\quad \vec{c}] = 144$

$\Rightarrow \frac{1}{6}\left(\frac{20+a^2}{2}\right)\left(\frac{20+a^2}{a}\right)\left(\frac{20+a^2}{4}\right) = 144$

$\Rightarrow (20 + a^2)^3 = 144 \times 48a$

$\Rightarrow a = 2$

$\therefore$ equation of plane $: 2x + 2y + 4z = 24$

$\Rightarrow x + y + 2z = 12$

$(0,3,4)$ does not lie on the plane.

7. If $z = \frac{i-1}{\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}}$, then z is:

A. $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$
B. $\frac{1}{\sqrt{2}}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$
C. $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$
D. $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

Answer (C)

Solution:

$z = \frac{i-1}{\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}}$

$= \frac{i-1}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}$

$= \frac{2(i-1)}{(1+i\sqrt{3})} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$

$= \frac{1}{2}\left(\sqrt{3}-1\right) + \frac{i}{2}\left(\sqrt{3}+1\right)$

$\therefore Arg\ (z) = \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) = \frac{5\pi}{12}$

$|z| = \sqrt{2}$

$z = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

8. Given that $\theta \in [0,\ 2\pi]$, the largest interval of values of θ which satisfy the inequation $\sin^{-1}\sin\theta - \cos^{-1}\sin\theta \geq 0$ is:

A. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$
B. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$
C. $[0,\ \pi]$
D. $\left[\frac{\pi}{2}, \frac{5\pi}{4}\right]$

Answer (A)

Solution:

$\sin^{-1}\sin\theta - \left(\frac{\pi}{2} - \sin^{-1}\sin\theta\right) \geq 0$

$\Rightarrow \sin^{-1}\sin\theta \geq \frac{\pi}{4}$

$\Rightarrow \frac{1}{\sqrt{2}} \leq \sin\theta \leq 1$

$\Rightarrow \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

9. If $\left((p \wedge q) \Rightarrow (r \vee q)\right) \wedge \left((p \wedge r) \Rightarrow q\right)$ is tautology, where $r \in \{p,\ q,\ \sim p,\ \sim q\}$, then the number of values of r is

A. 1
B. 2
C. 3
D. 4

Answer (B)

Solution:

$\left((p \wedge q) \Rightarrow (r \vee q)\right) \wedge \left((p \wedge r) \Rightarrow q\right)$

$\Rightarrow \left((\sim p \vee \sim q) \vee (r \vee q)\right) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow \left(\sim p \vee r \vee (q \vee \sim q)\right) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow T \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow (\sim p \vee \sim r \vee q)$

For the above statements to be tautology

r can be $\sim p$ or q

∴ Two values of r is possible.

10. If $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$, Given that $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$,then find $\frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}}$

A. 3
B. $-\sqrt{3}$
C. 1
D. -3

Answer (B)

Solution:

$2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$

$2(\vec{a} \times \vec{b}) - 3(\vec{c} \times \vec{a}) = 0$

$2(\vec{a} \times \vec{b}) + 3(\vec{a} \times \vec{c}) = 0$

$\vec{a} \times (2\vec{b} + 3\vec{c}) = 0$

$\Rightarrow \vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$\Rightarrow |\vec{a}|^2 = \lambda^2 \left(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12\vec{b} \cdot \vec{c}\right)$

$\Rightarrow 31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$

$\Rightarrow \vec{a} = \pm(2\vec{b} + 3\vec{c})$

$|\vec{b} \times \vec{c}|^2 = |\vec{b}|^2 |\vec{c}|^2 - (\vec{b} \cdot \vec{c})^2$

$= \frac{1}{4} \times 4 - \left(1 \times \left(-\frac{1}{2}\right)\right)^2$

$|\vec{b} \times \vec{c}|^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$|\vec{b} \times \vec{c}| = \frac{\sqrt{3}}{2}$

$\frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}} = \frac{|(2\vec{b} + 3\vec{c}) \times \vec{c}|}{(2\vec{b} + 3\vec{c}) \cdot \vec{b}}$

$\frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}} = \frac{\sqrt{3}}{\frac{1}{2} + 3 \times 2 \times \frac{1}{2} \times \left(-\frac{1}{2}\right)}$

$\Rightarrow \frac{|\vec{a} \times \vec{c}|}{\vec{a} \cdot \vec{b}} = -\sqrt{3}$

11. Number of 7 digit odd no's formed using 7 digits $1, 2, 2, 2, 3, 3, 5$ will be:

A. 80
B. 420
C. 240
D. 140

Answer (C)

Solution:

Even number formed
$\_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ 2$
Number of ways $= \frac{6!}{2!2!} = 180$
Total number of ways $= \frac{7!}{3!2!} = \frac{720\times7}{12} = 420$
Odd numbers $= 420 - 180$
$= 240$

12. The minimum value of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$ where $[.]$ denotes greatest integer function, is:

A. $\frac{3}{4}$
B. $\frac{5}{4}$
C. $\frac{1}{4}$
D. 0

Answer (A)

Solution:
$g(x) = x^2 - x + 1$
$= x^2 - 2 \cdot x \cdot \frac{1}{2} + \frac{1}{4} - \frac{1}{4} + 1$
$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$
$g(x)$ attains minimum value when $x = \frac{1}{2}$
So $f(x)$ is minimum when $x = \frac{1}{2}$
$f\left(\frac{1}{2}\right) = \frac{3}{4} + 0$
$= \frac{3}{4}$

13. The foci and eccentricity of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $(\pm4, 0)$ and $\sqrt{3}$ respectively. Then the length of latus rectum of the hyperbola is:

A. 8
B. $\frac{16}{\sqrt{3}}$
C. 4
D. $2\sqrt{3}$

Answer (B)

Solution:

$ae = 4$

$\Rightarrow a = \frac{4}{\sqrt{3}}$

Length of LR= $\frac{2b^2}{a}$

$= \frac{2}{a}a^2(e^2 - 1)$

$= 2a(e^2 - 1)$

$= \frac{8}{\sqrt{3}}(3 - 1)$

$= \frac{16}{\sqrt{3}}$

14. If $[\alpha\ \beta\ \gamma]\begin{bmatrix} 5 & 6 & 8 \\ 6 & 3 & 8 \\ -1 & 3 & 0 \end{bmatrix} = [0\ 0\ 0]$ where $(\alpha,\ \beta,\ \gamma)$ be a point on $2x + 5y + 3z = 5$, then $6\alpha + 5\beta + 9\gamma$ is equal to:

A. 20

B. $\frac{20}{3}$

C. 21

D. 7

Answer (B)

Solution:

$5\alpha + 6\beta - \gamma = 0$

$6\alpha + 3\beta + 3\gamma = 0$

$8\alpha + 8\beta = 0$

$\Rightarrow \alpha = -\beta\ \&\ \beta = \gamma$

Let $\alpha = k,\ \beta = -k,\ \gamma = -k$

$(\alpha,\ \beta,\ \gamma)$ lie on $2x + 5y + 3z = 5$

$\Rightarrow 2(k) + 5(-k) + 3(-k) = 5$

$\Rightarrow k = -\frac{5}{6}$

$\Rightarrow \alpha = -\frac{5}{6},\ \beta = \frac{5}{6},\ \gamma = \frac{5}{6}$

$\therefore 6\alpha + 5\beta + 9\gamma = -5 + \frac{25}{6} + \frac{45}{6} = \frac{20}{3}$

15. Coefficient of x^{-6} in expansion of $\left(\frac{4x}{5} - \frac{5}{2x^2}\right)^9$ is:

Answer (−5040)

Solution:

$T_{r+1} = {}^9C_r \left(\frac{4x}{5}\right)^{9-r} \left(-\frac{5}{2x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^9C_r \left(\frac{4}{5}\right)^{9-r} \left(-\frac{5}{2}\right)^r x^{9-r-2r}$

For coefficient of x^{-6}: $9 - r - 2r = -6$

$\Rightarrow r = 5$

$\therefore$ Coefficient of $x^{-6} = {}^9C_5 \left(\frac{4}{5}\right)^4 \left(-\frac{5}{2}\right)^5$

$= \frac{9!}{5!4!} \times \frac{4^4}{5^4} \times \frac{(-5)^5}{2^5}$
$= -5040$

16. The value of sum $1 \cdot 1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + \cdots + 15 \cdot (29)^2$ is ______.

Answer (6952)

Solution:

$S = 1 \cdot 1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + \cdots + 15 \cdot (29)^2$
$S = 1 \cdot 1^2 + 2 \cdot 3^2 + 3 \cdot 5^2 + 4 \cdot 7^2 + \cdots + 15 \cdot (29)^2 - 2(2 \cdot 3^2 + 4 \cdot 7^2 + \cdots + 14 \cdot (27)^2)$
$S = \sum_{n=1}^{15} n(2n-1)^2 - 2\sum_{n=1}^{7}(2n)(4n-1)^2$
$= \sum_{n=1}^{15}(4n^3 - 4n^2 + n) - 4\sum_{n=1}^{7}(16n^3 - 8n^2 + n)$
$= \left[4 \times \left(\frac{15\times16}{2}\right)^2 - 4 \times \frac{(15)(16)(31)}{6} + \frac{15\times(16)}{2}\right] - \left[64 \times \left(\frac{7\times8}{2}\right)^2 - 32 \times \frac{7(8)(15)}{6} + 4 \times \frac{7(8)}{2}\right]$
$= (4(14400) - 4(1240) + 120) - (64(28)^2 - 140(32) + 112)$
$= 52760 - 45808 = 6952$

17. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 matrix such that $a, b, c, d \in \{0, 1, 2, 3, 4\}$. The number of matrices A such that sum of elements of A is a prime number lying between 2 and 13 is __________.

Answer (204)

Solution:

As given $a + b + c + d = 3$ or 5 or 7 or 11
If $a + b + c + d = 3$
x^3 coefficient in $(1 + x + x^2 + \cdots + x^4)^4$
x^3 coefficient in $(1 - x^5)^4(1 - x)^{-4}$
$= {}^{4+3-1}C_3 = {}^{6}C_3 = 20$ $[\because x^r$ coefficient in $(1-x)^{-n} = {}^{n+r-1}C_r]$
If $a + b + c + d = 5$
x^5 coefficient in $(1 - x^5)^4(1 - x)^{-4} = (1 - 4x^5 + \cdots)(1 - x)^{-4}$ $= {}^{4+5-1}C_5 - 4\,{}^{4+0-1}C_0 = {}^{8}C_5 - 4 = 52$
If $a + b + c + d = 7$
x^7 coefficient in $(1 - 4x^5)^4(1 - x)^{-4} = (1 - 4x^5 + \cdots)(1 - x)^{-4}$
$= {}^{4+7-1}C_7 - 4 \cdot {}^{4+2-1}C_2 = {}^{10}C_7 - 4 \cdot {}^{5}C_2 = 80$
If $a + b + c + d = 11$
x^{11} coefficient in in $(1 - x^5)^4(1 - x)^{-4} = (1 - 4x^5 + 6x^{10})(1 - x)^{-4}$

$= {}^{4+11-1}C_{11} - 4 \cdot {}^{4+6-1}C_6 + 6 \cdot {}^{4+1-1}C_1$
$= {}^{14}C_{11} - 4 \cdot {}^{9}C_6 + 6 \cdot 4 = 364 - 336 + 24 = 52$
Total matrices $= 20 + 52 + 80 + 52 = 204$

18. If $\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{11}{21}$, then $n^2 + n + 15$ equals ______.

Answer (45)

Solution:

$\frac{{}^{2n+1}P_{n-1}}{{}^{2n-1}P_n} = \frac{11}{21}$
$\Rightarrow \frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \frac{11}{21}$
$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$
$\Rightarrow \frac{2(2n+1)}{(n+1)(n+2)} = \frac{11}{21}$
$\Rightarrow 84n + 42 = 11n^2 + 33n + 22$

$\Rightarrow 11n^2 - 51n - 20 = 0$
$\Rightarrow 11n^2 - 55n + 4n - 20 = 0$
$\Rightarrow (11n + 4)(n - 5) = 0$
$\Rightarrow n = 5$ or $n = -\frac{4}{11}$ (not possible)
$\Rightarrow n = 5$
$\Rightarrow n^2 + n + 15 = 25 + 5 + 15 = 45$

19. $\int_0^\alpha \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \frac{16+20\sqrt{2}}{15}$ then α is equal to __________.

Answer (2)

Solution:

$\int_0^\alpha \frac{x}{\sqrt{x+\alpha}-\sqrt{x}}dx = \int_0^\alpha \frac{1}{\alpha}\left[(x+\alpha)^{\frac{3}{2}} - \alpha(x+x)^{\frac{1}{2}} + x^{\frac{3}{2}}\right]dx$
$= \frac{1}{\alpha}\left[\frac{2}{5}(x+\alpha)^{\frac{5}{2}} - \alpha\frac{2}{3}(x+\alpha)^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}}\right]_0^\alpha$
$= \frac{1}{\alpha}\left[\frac{2}{5}\cdot(2\alpha)^{\frac{5}{2}} - \frac{2\alpha}{3}(2\alpha)^{\frac{3}{2}} + \frac{2}{5}\alpha^{\frac{5}{2}} - \frac{2}{5}\alpha^{\frac{5}{2}} + \frac{2}{3}\alpha^{\frac{5}{2}}\right]$
$= \frac{1}{\alpha}\left(\frac{2^{\frac{7}{2}}\alpha^{\frac{5}{2}}}{5} - \frac{2^{\frac{5}{2}}\alpha^{\frac{5}{2}}}{3} + \frac{2}{3}\alpha^{\frac{5}{2}}\right)$
$= \alpha^{\frac{3}{2}}\left(\frac{2^{\frac{7}{2}}}{5} - \frac{2^{\frac{5}{2}}}{3} + \frac{2}{3}\right)$
$= \frac{\alpha^{\frac{3}{2}}}{15}\left(3\cdot 2^{\frac{7}{2}} - 5\cdot 2^{\frac{5}{2}} + 10\right)$
$= \frac{\alpha^{\frac{3}{2}}}{15}\left(24\sqrt{2} - 20\sqrt{2} + 10\right)$
$= \frac{\alpha^{\frac{3}{2}}}{15}\left(4\sqrt{2} + 10\right) = \frac{16+20\sqrt{2}}{15}$
$\Rightarrow \alpha = 2$